

A New Combination of Message Passing Techniques for Receiver Design in MIMO-OFDM Systems

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Abstract—In this paper, we propose a new combined message passing algorithm which allows belief propagation (BP) and mean field (MF) applied on a same factor node, so that MF can be applied to hard constraint factors. Based on the proposed message passing algorithm, a iterative receiver is designed for MIMO-OFDM systems. Both BP and MF are exploited to deal with the hard constraint factor nodes involving the multiplication of channel coefficients and data symbols to reduce the complexity of the only BP used. The numerical results show that the BER performance of the proposed low complexity receiver closely approach that of the state-of-the-art receiver, where only BP is used to handled the hard constraint factors, in the high SNRs.

Index Terms—message passing receiver, belief propagation, mean field.

I. INTRODUCTION

RECENTLY, multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is a key technology for many wireless communication standards, due to its high spectral efficiency [1]. Message passing techniques performing Bayesian inference on factor graphs [2] have proven to be a very useful tool to design receivers in communication systems. And there are several message passing based receivers [3]–[7] with joint channel estimation and decoding for MIMO-OFDM systems in literature.

Belief propagation (BP), also known as sum-product algorithm [8], is the most popular message passing technique for its excellent performance, especially applied in discrete probabilistic models. BP has been widely used to design iterative receivers in digital communications. Its remarkable performance, especially when applied to discrete probabilistic models, justifies its popularity. However, its complexity may become intractable in certain application contexts, e.g. when

the probabilistic model includes both discrete and continuous random variables. As an alternative to BP, variational methods based on the mean field (MF) approximation have been initially used in quantum and statistical physics. The MF approximation has also been formulated as a message passing algorithm, referred to as variational message passing (VMP) algorithm [9]. It has primarily been used on continuous probabilistic conjugate-exponential models.

Yedidia et. al. proposed to derive the fixed point equations of both BP and MF approximation by minimizing region-based free energy approximation in [10]. A unified message passing framework which combines BP with MF [11] is proposed based on a particular region-based free energy approximation. Combined BP-MF allows that one divides the factor nodes on a factor graph into two disjoint subsets: a BP part and a MF part. The messages passed to or outgoing from a factor node in the BP part are computed by BP rule, while MF rule for a factor node in the MF part. Therefore, it keeps the virtues of BP and MF but avoid their respective drawbacks. The MIMO-OFDM receivers [3], [6], [7] are proposed using combined BP-MF.

In this paper, we heuristically apply both BP- and MF-like rule for a same factor node when designing message passing receiver for MIMO-OFDM systems. Generally speaking, MF rule is not suitable for hard constraint factor nodes for the logarithm operation to the factor. We handle a special kind of hard factor nodes on a stretched factor graph representation [7] using BP-like rule, yielding an exponential function, and then MF-like rule becomes possible to be applied. A new message passing receiver with low complexity is obtained the hybrid message computation of a same factor node.

Notation- Boldface lowercase and uppercase letters denote vectors and matrices, respectively. The expectation operator with respect to a pdf $g(x)$ is expressed by $\langle f(x) \rangle_{g(x)} = \int f(x)g(x)dx / \int g(x')dx'$, while $\text{var}[x]_{g(x)} = \langle |x|^2 \rangle_{g(x)} - |\langle x \rangle_{g(x)}|^2$ stands for the variance. The pdf of a complex Gaussian distribution with mean μ and variance ν is represented by $\mathcal{CN}(x; \mu, \nu)$. The relation $f(x) = cg(x)$ for some positive constant c is written as $f(x) \propto g(x)$.

II. THE NEW COMBINED MESSAGE PASSING FRAMEWORK

As a theoretical unified message passing framework, combined BP-MF algorithm [11] classifies the factor nodes \mathcal{A} on a factor graph into a BP subset \mathcal{A}_{BP} and a MF subset \mathcal{A}_{MF} , fulfilling $\mathcal{A}_{\text{BP}} \cup \mathcal{A}_{\text{MF}} = \mathcal{A}$ and $\mathcal{A}_{\text{BP}} \cap \mathcal{A}_{\text{MF}} = \emptyset$. Then the messages passed on the factor graph is updated by the

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following equations,

$$m_{f_a \rightarrow x_i}^{\text{BP}}(x_i) = \langle f_a(\mathbf{x}_a) \rangle_{\prod_{j \in N(a) \setminus i} n_{x_j \rightarrow f_a}(x_j)}, \quad a \in \mathcal{A}_{\text{BP}}, i \in N(a) \quad (1)$$

$$m_{f_a \rightarrow x_i}^{\text{MF}}(x_i) = \exp \left\{ \langle \log f_a(\mathbf{x}_a) \rangle_{\prod_{j \in N(a) \setminus i} n_{x_j \rightarrow f_a}(x_j)} \right\}, \quad a \in \mathcal{A}_{\text{MF}}, i \in N(a) \quad (2)$$

$$n_{x_i \rightarrow f_a}(x_i) \propto \prod_{b \in \mathcal{A}_{\text{BP}}(i) \setminus a} m_{f_b \rightarrow x_i}^{\text{BP}}(x_i) \prod_{c \in \mathcal{A}_{\text{MF}}(i)} m_{f_c \rightarrow x_i}^{\text{MF}}(x_i), \quad i \in \mathcal{I}, a \in N(i) \quad (3)$$

where $N(a)$ and $N(i)$ denotes the subset of variable nodes neighboring the factor node f_a and the subset of factor nodes neighboring the variable node x_i , respectively, $\mathcal{A}_{\text{BP}}(i) = \mathcal{A}_{\text{BP}} \cap N(i)$ and $\mathcal{A}_{\text{MF}}(i) = \mathcal{A}_{\text{MF}} \cap N(i)$.

The pure message passing algorithms, such as BP and MF, and the combined BP-MF only allow one message update rule to handle a factor node. In this work, we heuristically propose a new combination, which allows one exploit both BP- and MF-like rule to deal with a single factor node, e.g. f in Fig. 1. To calculate the message from the factor node f to variable node y , a BP-like rule is used at first,

$$\tilde{f}(x, y) = \langle f(x, y, z) \rangle_{n_{z \rightarrow f_\delta}(z)}, \quad (4)$$

then a MF-like rule is applied,

$$m_{f_\delta \rightarrow y}(y) = \exp \left\{ \left\langle \log \tilde{f}(x, y) \right\rangle_{b(x)} \right\}. \quad (5)$$

The new combination is well suitable for the case where the factor f is a hard constraint, such as $f(x, y, z) = \delta(z - xy)$, and the message $n_{z \rightarrow f}(z)$ is of exponential form. We will apply it to design a low complexity receiver for MIMO-OFDM systems in the following sections.

III. SYSTEM MODEL OF MIMO-OFDM SYSTEMS

Consider the uplink of a multiuser MIMO-OFDM system which consists of a receiver equipped with M antennas and N users, each equipped with one antenna. To combat the inter-symbol interference, OFDM with K subcarriers is adopted. The transmitted symbols by the n th user in frequency domain are denoted by $\mathbf{x}_n = [x_n(1), \dots, x_n(K)]^T$. Among the K subcarriers, K_p uniformly spaced subcarriers are selected for the users to transmit pilot signals, and the set of pilot-subcarriers of user n is denoted by \mathcal{P}_n . As in [4], we assume that $\cap \mathcal{P}_n = \emptyset$, and when a pilot-subcarrier is employed by a user, the remaining users do not transmit signals at the pilot-subcarrier. By the introduction of auxiliary variables $z_{mnk} = x_{nk} h_{mnk}$ and $\tau_{mk} = \sum_n z_{mnk}$, the received signal by the m th receive antenna at the k th subcarrier can be written as

$$\begin{aligned} y_{mk} &= \sum_n h_{mnk} x_{nk} + \omega_m \\ &= \sum_n z_{mnk} + \omega_m = \tau_{mk} + \omega_m, \end{aligned} \quad (6)$$

where h_{mnk} stand for the frequency-domain channel weight between the n th transmit antenna and the m th receive antenna, and ω_m denotes the additive white Gaussian noise (AWGN)

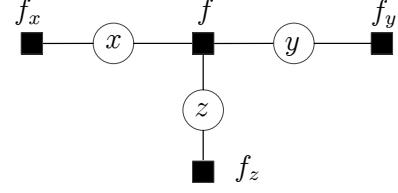


Fig. 1. A simple factor graph

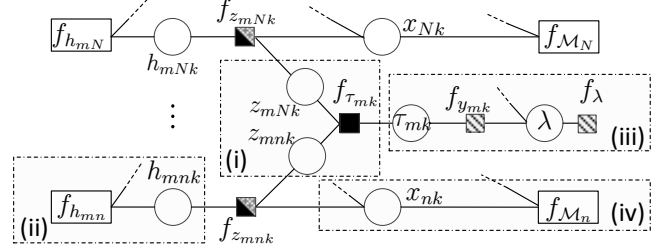


Fig. 2. A factor graph representation for the MIMO-OFDM system

with zero mean and variance λ^{-1} . The transmitted and received symbol by the n th user and m th receiver in frequency domain are denoted by x_{nk} and y_{mk} respectively.

From the receive model demonstrated in (6), the joint pdf of the collection of observed and unknown variables in the multi-signal model can be factorized as

$$p(\mathbf{y}, \boldsymbol{\tau}, \mathbf{z}, \mathbf{h}, \mathbf{x}, \lambda) = f_{\lambda}(\lambda) \prod_{m,n,k} f_{z_{mnk}}(z_{mnk}, x_{nk}, h_{mnk}) \times \prod_{m,k} f_{\tau_{mk}}(\tau_{mk}, \mathbf{z}_{mk}) f_{y_{mk}}(\lambda, \tau_{mk}) \prod_{m,n,k} f_{h_{mnk}} \prod_n f_{\mathcal{M}_n}, \quad (7)$$

where $f_{y_{mk}}(\lambda, \tau_{mk}) \triangleq \mathcal{CN}(y_{mk}; \tau_{mk}, \lambda^{-1})$ denotes the observation node, factors $f_{\tau_{mk}}(\tau_{mk}, \mathbf{z}_{mk}) = \delta(\tau_{mk} - \sum_n z_{mnk})$ and $f_{z_{mnk}}(z_{mnk}, x_{nk}, h_{mnk}) = \delta(z_{mnk} - x_{nk} h_{mnk})$ represents the constraint relationship of variables z_{mnk} , x_{nk} , h_{mnk} , and vector \mathbf{z}_{mk} denoted as $\mathbf{z}_{mk} = [z_{m1k}, \dots, z_{mNk}]^T$. The modulation, coding, and interleaving constraints are denoted by $f_{\mathcal{M}_n}$, and $f_{h_{mnk}}$ represents the priori of channel weight h_{mnk} . We assume that noise precision is unknown with the a priori $f_{\lambda} = 1/\lambda$. The above factorization lead to a factor graph representation shown in Fig. 2.

IV. RECEIVER DESIGN USING THE NEW COMBINED MESSAGE PASSING ALGORITHM

The difference between the proposed receiver and that in [7] lies in how to calculate the messages related to the factor nodes $\{f_{z_{mnk}}\}$. [7] handles the factor nodes $\{f_{z_{mnk}}\}$ by using BP rule, and expectation propagation (EP) is also exploit to covert some messages to be Gaussian. In this work, we adopt the new combination method to deal with the factor nodes. Therefore, the same messages in [7] will not be listed, and we only detailed the computation of the messages $n_{x_{nk} \rightarrow f_{\mathcal{M}_n}}(x_{nk})$ for soft demodulation, $n_{h_{mnk} \rightarrow f_{h_{mnk}}}(h_{mnk})$ for channel estimation and $m_{f_{z_{mnk}} \rightarrow z_{mnk}}(z_{mnk})$ for multi-signal interference cancellation. In addition, we assume that the messages $m_{f_{h_{mnk}} \rightarrow h_{mnk}}(h_{mnk}) = \mathcal{CN}(h_{mnk}; \bar{h}_{mnk}, \bar{v}_{h_{mnk}})$,

$m_{f_{\mathcal{M}_n} \rightarrow x_{nk}}(x_{nk}) = \sum_s \gamma_{nk}^s \delta(x_{nk} - s)$ and $n_{z_{mnk} \rightarrow f_{z_{mnk}}}(z_{mnk}) = \mathcal{CN}(z_{mnk}; \tilde{z}_{mnk}, \tilde{\nu}_{z_{mnk}})$ are known, and have listed in [7].

A. The Message for Soft Demodulation

At first, we apply a BP-like rule to the hard constraint factor node $f_{z_{mnk}}$, yielding

$$\begin{aligned} \tilde{f}_{z_{mnk}}(x_{nk}, h_{mnk}) &= \langle f_{z_{mnk}}(z_{mnk}, x_{nk}, h_{mnk}) \rangle_{n_{z_{mnk} \rightarrow f_{z_{mnk}}}(z_{mnk})} \\ &= \mathcal{CN}(x_{nk} h_{mnk}; \tilde{z}_{mnk}, \tilde{\nu}_{z_{mnk}}). \end{aligned} \quad (8)$$

Then, the message $m_{f_{z_{mnk}} \rightarrow x_{nk}}(x_{nk})$ is updated by a MF-like rule,

$$\begin{aligned} m_{f_{z_{mnk}} \rightarrow x_{nk}}(x_{nk}) &= \exp \left\{ \left\langle \log \tilde{f}_{z_{mnk}}(x_{nk}, h_{mnk}) \right\rangle_{b(h_{mnk})} \right\} \\ &\triangleq \mathcal{CN}(x_{nk}; \vec{x}_{mnk}, \vec{\nu}_{x_{mnk}}). \end{aligned} \quad (9)$$

where $b(h_{mnk}) = \mathcal{CN}(h_{mnk}; \hat{h}_{mnk}, \nu_{h_{mnk}})$ is presented later in (14), and

$$\vec{x}_{mnk} = \frac{\hat{h}_{mnk}^* \tilde{z}_{mnk}}{|\hat{h}_{mnk}|^2 + \nu_{h_{mnk}}}, \quad \vec{\nu}_{x_{mnk}} = \frac{\tilde{\nu}_{z_{mnk}}}{|\hat{h}_{mnk}|^2 + \nu_{h_{mnk}}}. \quad (10)$$

The message $n_{x_{nk} \rightarrow f_{\mathcal{M}_n}}(x_{nk})$, passed to soft demodulation, is calculated by

$$\begin{aligned} n_{x_{nk} \rightarrow f_{\mathcal{M}_n}}(x_{nk}) &= \prod_{m,k} m_{f_{z_{mnk}} \rightarrow x_{nk}}(x_{nk}) \\ &\triangleq \mathcal{CN}(x_{nk}; \hat{\xi}_{nk}, \nu_{\xi_{nk}}) \end{aligned} \quad (11)$$

where

$$\nu_{\xi_{nk}} = \left(\sum_m \frac{1}{\vec{\nu}_{x_{mnk}}} \right)^{-1}, \quad \hat{\xi}_{nk} = \nu_{\xi_{nk}} \sum_m \vec{x}_{mnk}.$$

To update the message passed to channel estimation part, we have to calculate the belief of x_{nk} , given as

$$b(x_{nk}) = \frac{n_{x_{nk} \rightarrow f_{\mathcal{M}_n}}(x_{nk}) m_{f_{\mathcal{M}_n} \rightarrow x_{nk}}(x_{nk})}{\int_{x_{nk}} n_{x_{nk} \rightarrow f_{\mathcal{M}_n}}(x_{nk}) m_{f_{\mathcal{M}_n} \rightarrow x_{nk}}(x_{nk})}. \quad (12)$$

Its mean and variance are

$$\begin{aligned} \hat{x}_{nk} &= \langle x_{nk} \rangle_{b(x_{nk})} \\ \nu_{x_{nk}} &= \langle |x_{nk}|^2 \rangle_{b(x_{nk})} - |\hat{x}_{nk}|^2. \end{aligned}$$

B. The Message for Channel Estimation

Similar to Eq. (9), the message $m_{f_{z_{mnk}} \rightarrow h_{mnk}}(h_{mnk})$ is also updated by a MF-like equation,

$$\begin{aligned} m_{f_{z_{mnk}} \rightarrow h_{mnk}}(h_{mnk}) &= \exp \left\{ \left\langle \log \tilde{f}_{z_{mnk}}(x_{nk}, h_{mnk}) \right\rangle_{b(x_{nk})} \right\} \\ &\propto \mathcal{CN}(h_{mnk}; \tilde{h}_{mnk}, \tilde{\nu}_{h_{mnk}}) \end{aligned} \quad (13)$$

where

$$\tilde{h}_{mnk} = \frac{\hat{x}_{nk}^* \tilde{z}_{mnk}}{|\hat{x}_{nk}|^2 + \nu_{x_{nk}}}, \quad \tilde{\nu}_{h_{mnk}} = \frac{\tilde{\nu}_{z_{mnk}}}{|\hat{x}_{nk}|^2 + \nu_{x_{nk}}}.$$

The message for channel estimation is $n_{h_{mnk} \rightarrow f_{h_{mnk}}}(h_{mnk}) = m_{f_{z_{mnk}} \rightarrow h_{mnk}}(h_{mnk})$.

The belief of h_{mnk} is calculated as

$$\begin{aligned} b(h_{mnk}) &= n_{h_{mnk} \rightarrow f_{h_{mnk}}}(h_{mnk}) m_{f_{h_{mnk}} \rightarrow h_{mnk}}(h_{mnk}) \\ &\triangleq \mathcal{CN}(h_{mnk}; \hat{h}_{mnk}, \nu_{h_{mnk}}) \end{aligned} \quad (14)$$

where

$$\nu_{h_{mnk}} = \left(\tilde{\nu}_{h_{mnk}}^{-1} + \tilde{\nu}_{h_{mnk}}^{-1} \right)^{-1} \quad (15)$$

$$\hat{h}_{mnk} = \nu_{h_{mnk}} (\tilde{h}_{mnk} / \tilde{\nu}_{h_{mnk}} + \tilde{h}_{mnk} / \tilde{\nu}_{h_{mnk}}). \quad (16)$$

C. The Message for Multi-signal Interference Elimination

Since the factor node $f_{z_{mnk}}$ stands for the hard constraint $z_{mnk} = x_{nk} h_{mnk}$, the belief of $b(z_{mnk})$ can be equivalently computed as

$$b(z_{mnk}) = \langle f_{z_{mnk}}(z_{mnk}, x_{nk}, h_{mnk}) \rangle_{b(x_{nk}) b(h_{mnk})}$$

and its mean and variance are easily obtained

$$\hat{z}_{mnk} = \hat{x}_{nk} \hat{h}_{mnk} \quad (17)$$

$$\nu_{z_{mnk}} = |\hat{x}_{nk}|^2 \nu_{h_{mnk}} + |\hat{h}_{mnk}|^2 \nu_{x_{nk}} + \nu_{h_{mnk}} \nu_{x_{nk}}. \quad (18)$$

Then, the message $m_{f_{z_{mnk}} \rightarrow z_{mnk}}(z_{mnk})$ is calculated as ¹

$$\begin{aligned} m_{f_{z_{mnk}} \rightarrow z_{mnk}}(z_{mnk}) &= \frac{\text{Proj}_{\mathcal{G}} \{b(z_{mnk})\}}{n_{z_{mnk} \rightarrow f_{z_{mnk}}}(z_{mnk})} \\ &\triangleq \mathcal{CN}(z_{mnk}; \vec{z}_{mnk}, \vec{\nu}_{z_{mnk}}), \end{aligned}$$

where

$$\vec{\nu}_{z_{mnk}} = (1/\nu_{z_{mnk}} - 1/\tilde{\nu}_{z_{mnk}})^{-1} \quad (19)$$

$$\vec{z}_{mnk} = \vec{\nu}_{z_{mnk}} (\hat{z}_{mnk} / \nu_{z_{mnk}} - \tilde{z}_{mnk} / \tilde{\nu}_{z_{mnk}}). \quad (20)$$

V. SIMULATION RESULTS AND COMPLEXITY COMPARISON

Consider a MIMO-OFDM system with the same parameters as in [7]. We compare the proposed receiver with four state-of-the-art receivers in the literature²: (1)Aux: the auxiliary variable aided method proposed in [7]. (2)BP-MF: disjoint version of the receivers in [3]; (3) BP-MF-EPv: the receiver proposed in [6]; (4) BP-EP-GA: the BP-EP-based receiver in [4] with perfect noise precision. As a reference, the performance of the receiver with perfect channel weight \mathbf{h} , noise precision λ and multiuser interference cancellation is also included, denoted by matched filter bound (MFB).

Fig. 3 shows the BER performance of the different receivers with running 15 iterations. The proposed low complexity receiver has a performance loss of 0.5dB compared to the high complexity version, “Aux”, in the moderate Eb/N0. Meanwhile, it achieves a performance gain of more than 1dB compared to “BP-MF”, and outperforms “BP-EP-GA” and

¹For a pdf $b(x)$, $\text{Proj}_{\mathcal{G}} \{b(x)\} = \mathcal{CN}(x; m, \nu)$, where $m = \langle x \rangle_{b(x)}$ and $\nu = \langle |x|^2 \rangle_{b(x)} - |m|^2$, stands for projecting a function $b(x)$ to Gaussian family.

²For a fair comparison and explicitly demonstrating the virtues of the proposed receiver, all considered receivers update message $m_{f_{h_{mnk}} \rightarrow h_{mnk}}(h_{mnk})$ using the method in [7].

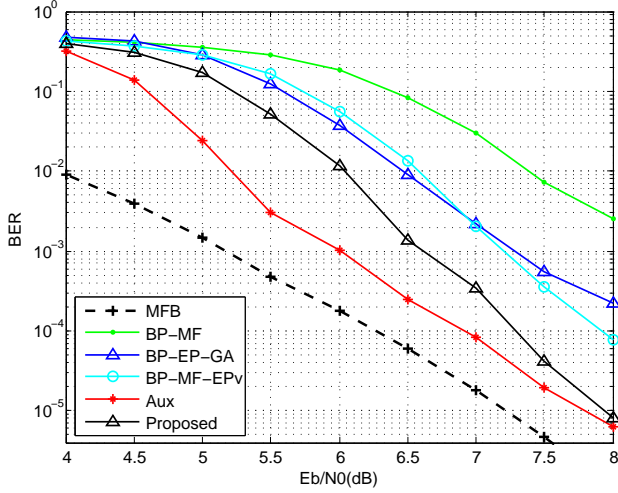


Fig. 3. BER performance of different algorithms.

“BP-MF-EPv” by about 0.5dB. Especially, the perform of proposed receiver can approach that of “Aux” at higher SNRs.

Since all the receivers employed the same method in the updating of messages $\{m_{f_{\mathcal{M}_n} \rightarrow x_{nk}}(x_{nk})\}$ and $\{n_{h_{mnk} \rightarrow f_{z_{mnk}}}(h_{mnk})\}$, here we only compare the complexity in handling the multiplication and multi-signal summation problem. The complexity of the proposed receiver, “BP-MF” and “BP-EP-GA” is in the order $\mathcal{O}(MNK + NKQ)$, since MN Gaussian message and NK discrete beliefs should be calculated, while the “BP-MF-EPv” has complexity of $\mathcal{O}(MN^2K + KN^3 + NKQ)$, where Q is the modulation order. Since MN Gaussian mixture belief should be computed, the complexity of “Aux” is $\mathcal{O}(MNKQ)$.

VI. CONCLUSION

In this paper, we propose a new combined message passing framework which will lead more flexible combination of BP and MF on factor graphs. It is applied to design a low complexity receiver for MIMO-OFDM systems. The receiver using the proposed message passing algorithm can obtain better trade-off between performance and complexity that the state-of-the-art receivers.

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